Sequences and Series

- Let a_1, a_2, a_3,…,a_n be a given sequence. Then, the expression

  $a_1 + a_2 + a_3 + \ldots a_n$ is called the series associated with the given sequence.

- The series $a_1 + a_2 + a_3 + \ldots a_n$ can be abbreviated as $\sum a_k$.
- A sequence a_1, a_2, a_3…, a_n… is called arithmetic sequence or arithmetic progression if $a_{n+1} = a_n + d$, $n \in N$, where a_1 is called the first term and the constant term d is called the common difference of the A.P.
- Then the nth term (general term) of the A.P. is $a_n = a + (n – 1) d$.

- Properties of AP

  - If a constant is added to each term of an A.P., the resulting sequence is also an A.P.
  - If a constant is subtracted from each term of an A.P., the resulting sequence is also an A.P.
  - If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.
  - If each term of an A.P. is divided by a non-zero constant then the resulting sequence is also an A.P.

- Sum upto n terms of an AP – $n/2[2a+(n-1)d]$
- Given two numbers a and b. We can insert a number A between them so that a, A, b is an A.P. Such a number A is called the arithmetic mean (A.M.) of the numbers a and b. $A = (a+b)/2$
- A sequence $a_1, a_2, a_3 \ldots, a_n$ is called geometric progression, if each term is non-zero and

  $a_{k+1}/a_k = r$(constant) for $k \geq 1$.

- General term of a GP $a_n = ar^{n-1}$
- Sum to n terms of a GP $S_n = a(1-r^n)/(1-r) = a(r^n-1)/(r-1)$.
- The geometric mean of two positive numbers a and b is the number $G = \sqrt{ab}$.
- $A = (a+b)/2$ and $G = \sqrt{ab}$

  $A - G = (a+b-2\sqrt{ab})/2$

  $(\sqrt{a} - \sqrt{b})^2 \geq 0$

  $A \geq G$.
- Sum of first n natural numbers = $n(n+1)/2$
- Sum of squares of first n natural numbers = $n(n+1)(2n+1)/6$.
- Sum of cubes of first n natural numbers = $[n(n+1)]^2/4$.

Sample Examples

- The income of a person is Rs 3,00,000, in the first year and he receives an increase of Rs.10,000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.

  Solution:-

  Here, we have an A.P. with a = 3,00,000, d = 10,000, and n = 20. Using the sum formula, we get

  $S_n = 20/2[600000+19*10000] = 7900000$

- Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.

  Solution:-

  Let A1, A2, A3, A4, A5 and A6 be six numbers between 3 and 24 such that 3, A1, A2, A3, A4, A5, A6, 24 are in A.P. Here, a = 3, b = 24, n = 8.

  Therefore, 24 = 3 + (8 –1) d, so that d = 3.

  Thus A1 = a + d = 3 + 3 = 6; A2 = a + 2d = 3 + 2 × 3 = 9;

  A3 = a + 3d = 3 + 3 × 3 = 12; A4 = a + 4d = 3 + 4 × 3 = 15;

  A5 = a + 5d = 3 + 5 × 3 = 18; A6 = a + 6d = 3 + 6 × 3 = 21.

- Find the sum to n terms of the series: 5 + 11 + 19 + 29 + 41…

  Solution:-

  $S_n = 5 + 11 + 19 + 29 + ... + a_{n-1} + a_n$

  $S_n = 5 + 11 + 19 + ... + a_{n-2} + a_{n-1} + a_n$

  On Subtraction,

  $0 = 5 + [6 + 8 + 10 + 12 + ...(n - 1) \text{ terms}] - a_n$

  $a_n = 5 + (n-1)[12+(n-2)*2]/2$

  $= 5 + (n - 1)(n + 4)$

  $= n^2 + 3n + 1$

$S_n = \sum a_k = \sum k^2+3k+1 = \sum k^2 + 3\sum k+1$ k varies from 1 to n

$= n(n+1)(2n+1)/6 + 3n(n+1)/2.$

- If A.M. and G.M. of two positive numbers a and b are 10 and 8, respectively, find the numbers.

  Solution:-

  $A = (a+b)/2 = 10 \quad G = \sqrt{ab} = 8$

  $a + b = 20$

  $a * b = 64$

  $(a - b)^2 = (a + b)^2 - 4ab$

  $(a - b)^2 = (20)^2 - 4*64$

  $(a - b) = \pm 12$

  The numbers a and b are 4, 16 or 16, 4 respectively.